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Optimization of Dynamic Systems in Industrial Applications

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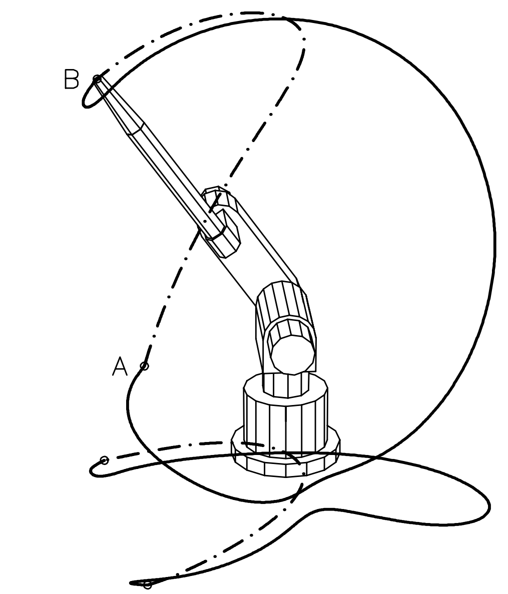
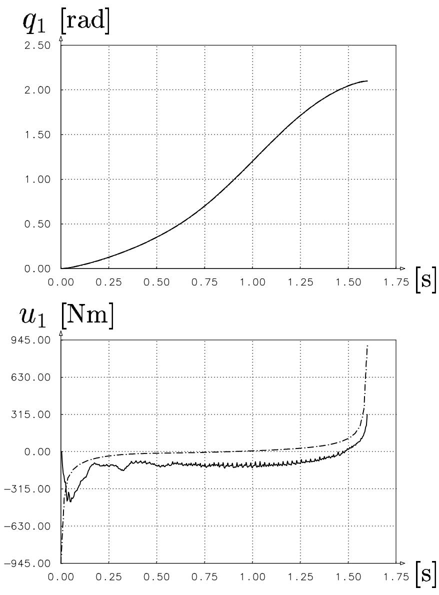
The simulation of technical processes by scientific computing has become an important tool for the development of new technologies. In many applications, the required theoretical and experimental research can be replaced in part by numerical computations. In this paper, we focus on technical processes which can be described by dynamic systems, i. e., by the solution of initial or boundary value problems in ordinary differential or differential algebraic equations. In this paper, several recently developed efficient numerical methods are presented. Their impact in engineering is demonstrated by their application to three different classes of problems, namely trajectory optimization, parameter estimation and design optimization.

P1. Optimal path planning for industrial robots



Minimum time motion in 1.322 s (———).
Minimum energy motion in 1.600 s (–·–·–).



Angle q_1 and control u_1 of the first joint (base) in simulation (–·–·–) and experiment (———) for the minimum energy motion (cf. left figure).

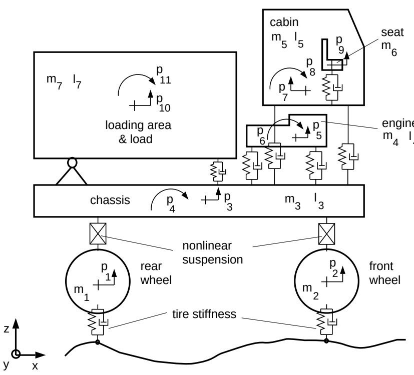
Figure 1: An example of robot trajectory optimization.

The application of numerical optimization methods to trajectory optimization for industrial robots has shown that large improvements are possible compared with traditional path planning methods (e. g., [9], [15]). But the use of sophisticated numerical optimization methods requires several necessities as, e. g., the proper description of the dynamic behavior of the robot as an n-body system (in minimal coordinates, e. g., the relative angles $q = (q_1, \ldots, q_n)^T$ between successive joints of the robot)

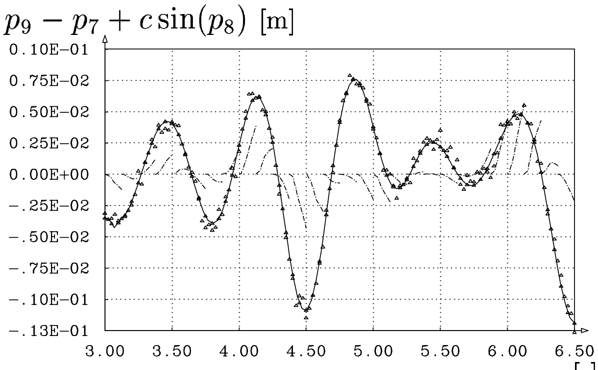
$$M(q(t)) \cdot \ddot{q}(t) \;=\; u(t) + \chi(\dot{q}(t), q(t)), \quad t \in [0, t_f], \tag{1}$$

where $u = (u_1(t), \ldots, u_n(t))^T$ is the torque control, M is the positive definite and symmetric $(n \times n)$-matrix of moments of inertia and χ are the moments resulting from centrifugal, coriolis, gravitational and frictional forces. Thus the first basic problem is the **modelling** of the dynamic system (1) of the robot [16]. The **identification** of unknown dynamic parameters of the specific robot (as, e. g., moments of inertia or friction parameters) is a second must (see problem P2). Once the dynamic equations have been modelled and determined in a proper way, the **optimization of trajectories** can be investigated. Different tasks for robots require different objectives for optimal trajectories, e. g., in welding optimal tracking of the prescribed path might be required. Otherwise if only the initial and the final position of the robot are prescribed a fast point-to-point movement might be requested. It has been demonstrated that the fastest point-to-point motion ($t_f \to$ min!) exhibits quite often a surprisingly structure, impacts enormous stress on the links and can even often not be realized in practice. Here, fast minimum energy motions ($\int_0^{t_f} \sum_{i=1}^{n} u_i^2(t)\,\mathrm{d}t \to$ min!) offer a compromise between time and stress [15]. The robot trajectories also have to satisfy several constraints as, e.,g., constraints on the controls, the angles and the angular velocities. Further constraints result from the geometrical design of the robot's working cell and require an efficient modelling of collision avoidance.

P2. Parameter identification in robotic and vehicle dynamics



Model of a truck [12].



Measurements and solution curve of the vertical motion of the driver's seat in relation to the cabin [7]

Figure 2: An example of parameter identification in vehicle dynamics.

The dynamic behavior of robots or vehicles can be described as **multibody systems** (after an index reduction) by

$$\begin{array}{rcl} \dot{y}(t) & = & f(t, y(t), u(t), p),\; t \in [0,1], \\ 0 & = & g(t, y(t), u(t), p), \\ 0 & = & r(y(0), u(0), y(1), u(1), p), \end{array} \quad y = \begin{pmatrix} y_1 \\ \vdots \\ y_{n_y} \end{pmatrix},\; f = \begin{pmatrix} f_1 \\ \vdots \\ f_{n_y} \end{pmatrix},\; u = \begin{pmatrix} u_1 \\ \vdots \\ u_{n_u} \end{pmatrix}, \tag{2}$$

$g = (g_1, \ldots, g_{n_u})^T$. For a sufficiently accurate simulation of the real system the knowledge of specific data is required as, e. g., for robots the moments of inertia and friction parameters, and for vehicles the damping coefficients.

Within a controlled experiment functions h_i, $i = 1, \ldots, n_h$, of the state variables (and also the input functions of the system) are measured at discrete t_j, $j = 1, \ldots, l$,

$$z_{i,j} = z_i(t_j) = h_i(t_j, y(t_j), u(t_j), p) + \epsilon_{i,j},\ i \in I_j \subset \{1, \ldots, n_h\},\ 0 \leq t_1 < t_2 < \ldots < t_l \leq 1, \quad (3)$$

where $\epsilon_{i,j}$ is the (unknown) measurement error. The task is to estimate p in such a way that the experiment that is simulated by a numerical integration of (2) does optimally fit the measurements. As a criterion for optimality the weighted **nonlinear least squares** objective

$$\ell_2(p, y(0), u(0)) = \sum_{i \in I_j,\ j=1,\ldots,l} \frac{(z_{i,j} - h_i(t_j, y(t_j), u(t_j), p))^2}{\omega_{i,j}^2} \longrightarrow \min_{p, y(0), u(0)} ! \qquad (4)$$

with $\omega_{i,j} =$ const. can be used, where $(y(t),\ u(t))$ is the solution of (2) for the parameters p and the initial values $y(0),\ u(0)$ that might be unknown, too.

P3. Optimum design of high frequent oscillators with minimized phase noise

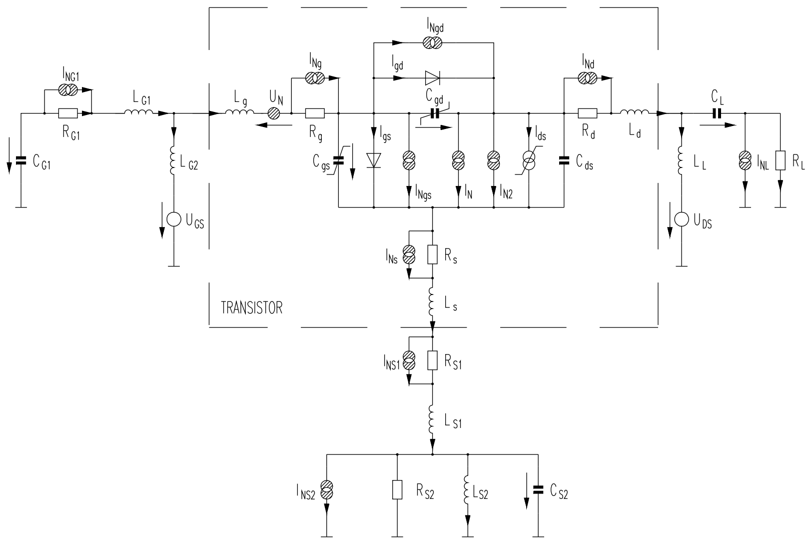


Figure 3: Equivalent circuit of the oscillator to be optimized [1].

A low **phase noise** is besides the signal properties essential to the design of **oscillating electrical circuits**. A new and general method to minimize the single-sideband phase noise of free running oscillators reduces the phase noise without requiring additional elements or the manufacture of prototypes. It is based on the description of the signal and noise behavior of an oscillator circuit by the Langevin equations

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \mathbf{f}(\mathbf{x}(t), \boldsymbol{\xi}(t), y(t)) = \mathbf{f}(\mathbf{x}(t)) + \mathbf{G}(\mathbf{x}(t)) \cdot \boldsymbol{\xi}(t) + \mathbf{g}(\mathbf{x}(t)) \cdot y(t) + \mathcal{O}(\boldsymbol{\xi}^2, y^2, \boldsymbol{\xi} y), \quad (5) \\ \mathbf{G}(\mathbf{x}(t)) &= \frac{\partial \mathbf{f}(\mathbf{x}(t), \boldsymbol{\xi}(t), y(t))}{\partial \boldsymbol{\xi}}\Big|_{\boldsymbol{\xi}=0, y=0}, \qquad \mathbf{g}(\mathbf{x}(t)) = \frac{\partial \mathbf{f}(\mathbf{x}(t), \boldsymbol{\xi}(t), y(t))}{\partial y}\Big|_{\boldsymbol{\xi}=0, y=0}, \end{aligned}$$

where $\mathbf{x}$ are the state variables of the circuit, $\boldsymbol{\xi}$ are the white noise sources and y is a nonlinear $f^{-\alpha}$ noise source denoting the baseband noise. The notation $\mathbf{f}(\mathbf{x}) := \mathbf{f}(\mathbf{x}, \mathbf{0}, 0)$ is used. The terms of order $\mathcal{O}(\boldsymbol{\xi}^2, y^2, \boldsymbol{\xi} y)$ are neglected. The single-sideband phase noise $L(f_m)$ can be simulated by solving (5) with a perturbation theory [8]

$$\begin{aligned} L(f_m) &= \frac{\Delta f_{3dB}}{\pi f_m^2} + \omega_0^2 |g_{1,0}|^2 \frac{c}{|2\pi f_m|^{2+\alpha}} \qquad (6) \\ \text{with} \quad \Delta f_{3dB} &= \frac{1}{4\pi} \omega_0^2 \frac{1}{T_0} \int_0^{T_0} \mathbf{v}(\mathbf{x}(t))^T \mathbf{G}(\mathbf{x}(t)) \boldsymbol{\Gamma}(\mathbf{x}(t)) \mathbf{G}(\mathbf{x}(t))^T \mathbf{v}(\mathbf{x}(t)) \mathrm{d}t \end{aligned}$$

$$\text{and} \quad g_{1,0} = \frac{1}{T_0} \int_0^{T_0} \mathbf{v}(\mathbf{x})^T \mathbf{g}(\mathbf{x}) \mathrm{d}t.$$

The first term on the right hand side of Eq. (6) describes the phase noise caused by the white noise sources. $\mathbf{v}(\mathbf{x}(t))$ is the left-sided eigenvector of the fundamental matrix $\mathbf{\Psi}(T_0, 0)$. The matrix $\mathbf{\Gamma}$ denotes the correlation matrix of the white noise sources. The second term of Eq. (6) describes the phase noise caused by the baseband noise. $g_{1,0}$ is a coefficient that characterizes the upconversion of the baseband noise to the carrier frequency. The modulation of the $f^{-\alpha}$ noise source due to the oscillation is taken into account as well as the upconversion of the baseband noise caused by the nonlinearities in the circuit. The factor c is derived from baseband noise measurements. The functions $\mathbf{x}(t)$ and $\mathbf{v}(t)$ depend on the design parameters $\mathbf{p}$ of the circuit by the system of nonlinear differential equations

$$\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), \mathbf{p}), \tag{7}$$

$$\dot{\mathbf{v}}(t) = -\left[\frac{\partial \mathbf{f}(\mathbf{x}(t), \mathbf{p})}{\partial \mathbf{x}}\right]^T \cdot \mathbf{v}(t), \quad t \in [0, T_0] \tag{8}$$

and appropriate boundary conditions. The minimization of the phase noise $L(f_m)$ with respect to the design parameters $\mathbf{p}$ and subject to Eqs. (7) and (8) is an **optimal control problem** with $\mathbf{p}$ as control from a finite dimensional control space and $\mathbf{x}$ and $\mathbf{v}$ as state variables of the optimal control problem. The optimization problem is solved numerically by the **direct collocation method** [1], [14].

In an **experiment** the method is applied to minimize the single-sideband phase noise of a planar integrated free running microwave oscillator at 15 GHz [1]. The equivalent circuit of the oscillator is depicted in Fig. 3. In this special case, five design parameters of the linear network are optimized subject to a system of 20 highly nonlinear differential equations (7), (8). A prototype of the new designed oscillator has been manufactured. A reduction of 10 dB of the phase noise caused by the upconverted baseband noise is measured at a frequence deviation of 10 kHz [1].

Numerical methods for parameter identification and optimal control

Methods for the solution of optimal control problems are the recently developed and implemented direct collocation and direct shooting methods (cf. [3], [6], [13], [14]). These methods have shown to be efficient, reliable and robust in solving real-life problems. If very high accuraccy is required then the indirect multiple shooting method is the choice [4]. A survey of efficient methods is given in [11].
Efficient methods for the solution of the mentioned parameter identification problems are based on multiple shooting in combination with generalized Gauss-Newton- or adapted SQP-methods (cf. [2], [7]). These identification algorithms only require some functions of the states $y(t)$ and no derivatives to be measured in order to identify the unknown parameters p. In other widely used approaches it is necessary to measure not only the first but also the second time derivatives of the state variables $y(t)$. As this is often not practicable, artifical measurements of time derivatives have to be constructed.
The proposed methods for identification and optimal control use the same dynamic model. They can therefore be conviently used in combination.

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